2020
MATHEMATICS
[HONOURS]
Paper : V

Full Marks : 100 Time : 4 Hours

The figures in the right-hand margin indicate marks.
Candidates are required to give their answers in their own words as far as practicable.
Symbols and notations have their usual meanings.

1. Answer any **five** questions: 1×5=5

   a) State the principle of virtual work for a free rigid body.

   b) Define cone of friction.

   c) What do you mean by wrench?

   d) Define momentum ellipsoid at any point of a rigid body.

   e) State the principle of conservation of angular momentum under impulsive forces.

   f) Define centre of pressure of a plane lamina.

   g) What is centre of buoyancy?

   h) What do you mean by field of force?

2. Answer any **ten** questions: 2×10=20

   a) Three forces P, Q, R act along the sides BC, CA and AB respectively of a triangle ABC in the same sense and their resultant passes through the in-centre, then show that P+Q+R=O.

   b) If a system consist of two forces, one of which act along OZ and mix components of the system are X, Y, Z; L, M, N, then show that the force along OZ is $\frac{LX + MY + NZ}{N}$.

   c) Which forces appear in the equation of virtual work?

   d) State the law of limiting friction.

   e) Find the moment of intertia of a hollow sphere about a diameter, its interval and external radii are a and b respectively.

   f) Masses 2, 3, 4, 5 and 7 are placed at the four angular points and centre of a square. Find the C. G. of the system.

   g) Find the time period of oscillation of a compound pendulum.

h) A circular plane rotates about an axis through its centre and perpendicular to its plane with an angular velocity ω. The axis is set free and a point in its circumference of the circular plane is fixed. Show that the resulting angular velocity is $\frac{\omega}{3}$.

i) A circular plane is immersed with its plane vertical in a liquid. Find the depth of its centre of pressure.

j) What are metacentric height and surface of buoyancy?

k) A triangle is immersed in a homogeneous liquid. Show that the sum of pressure at the vertices is three times the pressure at the centre.

l) A homogeneous fluid is in equilibrium under gravity only. Show that the surface of equal pressure are horizontal planes.

3. Answer any **five** questions: 6×5=30

a) An imperfectly rough sphere moves from rest down a plane inclined at an angle α to the horizon. Discuss the motion.

b) A bent lever, whose arms are of lengths a and b, the angle between them being α, makes a small oscillations in its own plane about the fulcrum, show that the length of the corresponding simple pendulum is $\frac{2}{3}\frac{a^3+b^3}{\sqrt{a^4+2a^2b^2\cos\alpha+b^4}}$.

c) Establish the energy test of stability of equilibrium of a rigid body and explain it for one degree for freedom.

d) Show that three coplanar forces P, Q, R acting at the points A, B, C are in astatic equilibrium if they meet at a point on the circum circle of the triangle ABC and if P : Q : R=a : b : c, where a, b, c are the sides of the triangle ABC.

e) Forces P, Q, R act along the straight lines y=b, z = –c; z = c, x = –a; x = a, y = –b respectively. Show that they will have a single resultant if $\frac{a}{P}+\frac{b}{Q}+\frac{c}{R}=O$. Also find the equation of the line of single resultant.

f) Define pressure at a point in a fluid. Show that the pressure at any point in a fluid at rest is the same in all directions. Is this true for fluids in motion?

g) A liquid filled the lower half of a circular tube of radius a in a vertical plane. If the tube is now rotated about the vertical diameter with uniform angular velocity ω such that the liquid is just separate into two parts, show that $\omega = \sqrt{\frac{2g}{a}}$.

h) If an area is bounded by two concentric semi-circles with their common bounding diameter in the free surface, prove that the depth of the centre of pressure is $\frac{3\pi}{16}\frac{(a+b)(a^2+b^2)}{(a^2+ab+b^2)}$, where a, b are radii of the semicircles and a>b.

Answer any **three** questions: 15×3=45

4. a) Two uniform similar rods (of same material) PQ and QT of length $2l$ and $2l'$ respectively– are rigidly united at Q and suspended from P. If they inclined at an angle α and β respectively to the vertical, prove that $(l^2+2ll')\sin\alpha = l'\sin\beta$. 7

b) Two equal forces act along the generations of the system of the hyperboloid $\frac{x^2+y^2}{a^2}-\frac{z^2}{b^2}=1$, and cut the plane z = 0 at the extremities of

perpendicular diameters of the circle $x^2+y^2=a^2$. Show that the pitch of the equivalent wrench is $\frac{a^2b}{a^2+2b^2}$. 8

5. a) A heavy hemispherical shell of radius r has a particle attached to a point on the rim and rests with the curved surface in contact with a sphere of radius R at the highest point. Prove that if $\frac{R}{r} > (\sqrt{5}-1)$ the equilibrium is stable, whenever be the weight of the particle. 7

b) Obtain the equation of the central axis for a given system of forces acting on a rigid body. 8

6. a) A fine glass tube in the shape of an equilateral triangle is filled with equal volumes of two liquids which do not mix, whose densities are in A.P. The tube is held in vertical plane and the side that contain the heaviest and lightest liquids makes angle θ with the verticle. Show that the surface of separation divide the sides the ratio $\cos\left(\frac{\pi}{6}-\theta\right):\cos\left(\frac{\pi}{6}+\theta\right)$. 7

b) A spherical vessel is just filled with a heavy liquid, the particle of which attract one another according to the law of gravitation. If the pressure at the highest point vanishes, show that the resultant thrust across a vertical plane is $\pi g\rho a^3 + \frac{1}{3}\pi^2\rho^2\gamma a^4$ where a is the radius, ρ is the density and γ is the constant of gravitation. 8

7. a) A solid circular cylinder of radius a rotating about its axis is placed gently with axis horizontal on a rough plane, whose inclination to the horizon is α. Initially the friction acts up the plane and the coefficient of friction is μ. Show that the cylinder will move upwards if $\mu > \tan\alpha$. Also show that the time that elapses before rolling commences is $\frac{a\omega}{g(3\mu\cos\alpha - \sin\alpha)}$, where ω is the initial angular velocity of the cylinder. 8

b) A plane area is immersed in a fluid of given density. Find the co-ordinates of centre of pressure. Also prove that the co-ordinates of centre of pressure remains same if it rotates through an angle θ about the line of intersection with the effective surface. 7

8. a) Establish the principle of independence of motion due to translation and rotation of a rigid body. 7

b) A rod of length 2a is suspended by a string of length l, attached to one end. It the string and the rod revolve with uniform angular velocity about a vertical and their inclination with the vertical be θ and φ respectively, then prove that $\frac{3l}{a}\frac{(4\tan\theta - 3\tan\varphi)\sin\varphi}{(\tan\varphi - \tan\theta)\sin\theta}$. 8

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